

## Binomial exponential sums

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**Abstract.** We obtain new bounds of exponential sums modulo a prime  $p$  with binomials  $ax^k + bx^n$ . In particular, for  $k = 1$ , we improve the bound of Karatsuba (1967) from  $O(n^{1/4} p^{3/4})$  to  $O\left(p^{3/4} + n^{1/3} p^{2/3}\right)$  for any  $n$ , and then use it to improve the bound of Akulinichev (1965) from  $O(p^{5/6})$  to  $O(p^{4/5})$  for  $n|(p-1)$ . The result is based on a new bound on the number of solutions and of degrees of irreducible components of certain equations over finite fields.

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