Moment varieties of measures on polytopes

KATHLÉN KOHN, BORIS SHAPIRO AND BERND STURMFELS

Abstract. The uniform probability measure on a convex polytope induces piece-wise polynomial densities on its projections. For a fixed combinatorial type of simplicial polytopes, the moments of these measures are rational functions in the vertex coordinates. We study projective varieties that are parametrized by finite collections of such rational functions. Our focus lies on determining the prime ideals of these moment varieties. Special cases include Hankel determinantal ideals for polytopal splines on line segments, and the relations among multisymmetric functions given by the cumulants of a simplex. In general, our moment varieties are more complicated than in these two special cases. They offer challenges for both numerical and symbolic computing in algebraic geometry.