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Sign-changing blowing-up solutions for the critical nonlinear heat equation

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Abstract. Let Ω be a smooth bounded domain in $\mathbb{R}^n$ and denote the regular part of the Green function on Ω with Dirichlet boundary condition by $H(x, y)$. Assume the integer k_0 is sufficiently large, $q \in \Omega$ and $n \geq 5$. For $k \geq k_0$ we prove that there exist initial data u_0 and smooth parameter functions $\xi(t) \to q$ and $0 < \mu(t) \to 0$ for $t \to +\infty$ such that the solution u_q of the critical nonlinear heat equation

$$\begin{cases} u_t = \Delta u + |u|^{\frac{4}{n-2}} u & \text{in } \Omega \times (0, \infty) \\ u = 0 & \text{on } \partial\Omega \times (0, \infty) \\ u(\cdot, 0) = u_0 & \text{in } \Omega \end{cases}$$

has the form

$$u_q(x, t) \approx \mu(t)^{-\frac{n-2}{2}} \left(Q_k \left(\frac{x - \xi(t)}{\mu(t)} \right) - H(x, q) \right),$$

where the profile Q_k is the non-radial sign-changing solution of the Yamabe equation

$$\Delta Q + |Q|^{\frac{4}{n-2}} Q = 0 \text{ in } \mathbb{R}^n,$$

constructed in [9]. In dimension 5 and 6 we also investigate the stability of $u_q(x, t)$.

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