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Moduli of non-standard Nikulin surfaces in low genus

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Abstract. Primitively polarized genus g Nikulin surfaces (S, M, H) are of two types, that we call standard and non-standard depending on whether the lattice embedding $\mathbb{Z}[H] \oplus_\perp \mathbf{N} \subset \mathrm{Pic}\, S$ is primitive. Here H is the genus g polarization and $\mathbf{N}$ is the Nikulin lattice. We concentrate on the non-standard case, which only occurs in odd genus. In particular, we study the birational geometry of the moduli space of non-standard Nikulin surfaces of genus g and prove its rationality for $g = 7, 11$ and the existence of a rational double cover of it when $g = 9$. Furthermore, if (S, M, H) is general in the above moduli space and $(C, M|_C)$ is a general Prym curve in $|H|$, we determine the dimension of the family of non-standard Nikulin surfaces of genus g containing $(C, M|_C)$ for $3 \leq g \leq 11$; this completes the study of the Prym-Nikulin map initiated in [11].

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