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Arnoux-Rauzy interval exchanges

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Abstract. The Arnoux-Rauzy systems are defined in [6], both as symbolic systems on three letters and exchange transformations of six intervals on the circle. In connection with a conjecture of S.P. Novikov, we investigate the dynamical properties of these interval exchange transformations, and precise their relation with the symbolic systems, which was known only to be a semi-conjugacy. In order to do this, we define a new system which is an exchange transformation of nine intervals on the line (it was described in [4] for a particular case). Our main result is that the semi-conjugacy determines a measure-theoretic isomorphism (between the three systems) under a diophantine (sufficient) condition, which is satisfied by almost all Arnoux-Rauzy systems for a suitable measure. However, under another condition, the interval exchange transformations are not uniquely ergodic and the isomorphism does not hold for all invariant measures. Finally, we give conditions for these interval exchange transformations to be weakly mixing.

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