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Approximation of holomorphic functions in Banach spaces admitting a Schauder decomposition

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Abstract. Let X be a complex Banach space. Recall that X admits a *finite-dimensional Schauder decomposition* if there exists a sequence $\{X_n\}_{n=1}^{\infty}$ of finite-dimensional subspaces of X, such that every $x \in X$ has a unique representation of the form $x = \sum_{n=1}^{\infty} x_n$, with $x_n \in X_n$ for every n. The finite-dimensional Schauder decomposition is said to be *unconditional* if, for every $x \in X$, the series $x = \sum_{n=1}^{\infty} x_n$, which represents x, converges unconditionally, that is, $\sum_{n=1}^{\infty} x_{\pi(n)}$ converges for every permutation π of the integers. For short, we say that X admits an unconditional F.D.D.

We show that if X admits an unconditional F.D.D. then the following Runge approximation property holds:

(R.A.P.) *There is $r \in (0, 1)$ such that, for any $\epsilon > 0$ and any holomorphic function f on the open unit ball of X, there exists a holomorphic function h on X satisfying $|f - h| < \epsilon$ on the open ball of radius r centered at the origin.*

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