

Higher arithmetic degrees of dominant rational self-maps

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Abstract. Suppose that $f: X \dashrightarrow X$ is a dominant rational self-map of a smooth projective variety defined over $\overline{\mathbf{Q}}$. Kawaguchi and Silverman conjectured that if $P \in X(\overline{\mathbf{Q}})$ is a point with well-defined forward orbit, then the growth rate of the height along the orbit exists, and coincides with the first dynamical degree $\lambda_1(f)$ of f if the orbit of P is Zariski dense in X .

In this note, we extend the Kawaguchi–Silverman conjecture to the setting of orbits of higher-dimensional subvarieties of X . We begin by defining a set of arithmetic degrees of f , independent of the choice of cycles, and we then develop the theory of arithmetic degrees in parallel to existing results for dynamical degrees. We formulate several conjectures governing these higher arithmetic degrees, relating them to dynamical degrees.

Mathematics Subject Classification (2020): 37P15 (primary); 14G40, 11G50, 32H50 (secondary) .