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Essential dimension and pro-finite group schemes

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Abstract. A. Vistoli observed that, if Grothendieck's section conjecture is true and X is a smooth hyperbolic curve over a field finitely generated over $\mathbb{Q}$, then $\underline{\pi}_1(X)$ should somehow have essential dimension 1. We prove that an infinite, pro-finite étale group scheme always has infinite essential dimension. We introduce a variant of essential dimension, the fce dimension, $\operatorname{fced} G$, of a pro-finite group scheme G, which naturally coincides with $\operatorname{ed} G$ if G is finite, but has a better behaviour in the pro-finite case. Grothendieck's section conjecture implies $\operatorname{fced} \pi_1(X) = \dim X = 1$ for X as above. We prove that, if A is an abelian variety over a field finitely generated over $\mathbb{Q}$, then $\operatorname{fced} \underline{\pi}_1(A) = \operatorname{fced} TA = \dim A$.

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