

## Almost formality of manifolds of low dimension

DOMENICO FIORENZA, KOTARO KAWAI, HÔNG VÂN LÊ  
AND LORENZ SCHWACHHÖFER

**Abstract.** In this paper we introduce the notion of Poincaré DGCA of Hodge type, which is a subclass of Poincaré DGCA encompassing the de Rham algebras of closed orientable manifolds. Then we introduce the notion of the small algebra and the small quotient algebra of a Poincaré DGCA of Hodge type. Using these concepts, we investigate the equivalence class of  $(r - 1)$ -connected ( $r > 1$ ) Poincaré DGCA of Hodge type. In particular, we show that an  $(r - 1)$ -connected Poincaré DGCA of Hodge type  $\mathcal{A}^*$  of dimension  $n \leq 5r - 3$  is  $A_\infty$ -quasi-isomorphic to an  $A_3$ -algebra and prove that the only obstruction to the formality of  $\mathcal{A}^*$  is a distinguished Harrison cohomology class  $[\mu_3] \in \text{Harr}^{3,-1}(H^*(\mathcal{A}^*), H^*(\mathcal{A}^*))$ . Moreover, the cohomology class  $[\mu_3]$  and the DGCA isomorphism class of  $H^*(\mathcal{A}^*)$  determine the  $A_\infty$ -quasi-isomorphism class of  $\mathcal{A}^*$ . This can be seen as a Harrison cohomology version of the Crowley-Nordström results [9] on rational homotopy type of  $(r - 1)$ -connected ( $r > 1$ ) closed manifolds of dimension up to  $5r - 3$ . We also derive the almost formality of closed  $G_2$ -manifolds, which have been discovered recently by Chan-Karigiannis-Tsang in [8], from our results and the Cheeger-Gromoll splitting theorem.

**Mathematics Subject Classification (2010):** 57R19 (primary); 53C25, 53C29, 58A10 (secondary).