

## Continuity of solutions to a basic problem in the calculus of variations

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**Abstract.** We study the problem of minimizing  $\int_{\Omega} F(Du(x)) dx$  over the functions  $u \in W^{1,1}(\Omega)$  that assume given boundary values  $\phi$  on  $\Gamma := \partial\Omega$ . The Lagrangian  $F$  and the domain  $\Omega$  are assumed convex. A new type of hypothesis on the boundary function  $\phi$  is introduced: the *lower* (or upper) *bounded slope condition*. This condition, which is less restrictive than the familiar bounded slope condition of Hartman, Nirenberg and Stampacchia, allows us to extend the classical Hilbert-Haar regularity theory to the case of semiconvex (or semiconcave) boundary data (instead of  $C^2$ ). We prove in particular that the solution is locally Lipschitz in  $\Omega$ . In certain cases, as when  $\Gamma$  is a polyhedron or else of class  $C^{1,1}$ , we obtain in addition a global Hölder condition on  $\bar{\Omega}$ .

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