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Defect of compactness for Sobolev spaces on manifolds with bounded geometry

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Abstract. *Defect of compactness*, relative to an embedding of two Banach spaces $E \hookrightarrow F$, is the difference between a weakly convergent sequence in E and its weak limit, taken up to a remainder that vanishes in the norm of F. For a number of known embeddings, Sobolev embeddings in particular, defect of compactness takes form of a *profile decomposition* - a sum of clearly structured terms with asymptotically disjoint supports, called *elementary concentrations*. In this paper we construct a profile decomposition for the Sobolev space $H^{1,2}(M)$ of a Riemannian manifold with bounded geometry, in the form of a sum of elementary concentrations associated with concentration profiles defined on manifolds induced by a limiting procedure at infinity, and thus different from M. The profiles satisfy an inequality of Plancherel type: the sum of the quadratic forms of Laplace-Beltrami operators for the profiles on their respective manifolds is bounded by the quadratic form of the Laplace-Beltrami operator of the sequence. A similar relation, related to the Brezis–Lieb Lemma, holds for the L^p-norms of profiles on the respective manifolds.

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