

## The smooth Riemannian extension problem

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**Abstract.** Given a metrically complete Riemannian manifold  $(M, g)$  with smooth non-empty boundary and assuming that one of its curvatures is subject to a certain bound, we address the problem of whether it is possible to realize  $(M, g)$  as a domain inside a geodesically complete Riemannian manifold  $(\bar{M}, \bar{g})$  without boundary, by preserving the same curvature bounds. In this direction we provide three kind of results: (1) a general existence theorem showing that it is always possible to obtain a geodesically complete Riemannian extension without curvature constraints; (2) various topological obstructions to the existence of a complete Riemannian extension with prescribed sectional and Ricci curvature bounds; (3) some existence results of complete Riemannian extensions with sectional and Ricci curvature bounds, mostly in the presence of a convexity condition on the boundary.

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