# Shi-type estimates of the Ricci flow based on Ricci curvature

CHIH-WEI CHEN

*Dedicated to Professor Gérard Besson on the occasion of his 60th birthday*

**Abstract.** We prove that the magnitude of the derivative of Ricci curvature can be uniformly controlled by the bounds of Ricci curvature and injectivity radius along the Ricci flow. As a consequence, a precise uniform local bound of curvature operator can be constructed from local bounds of Ricci curvature and injectivity radius among all $n$-dimensional Ricci flows. In particular, we show that every Ricci flow with $|\operatorname{Ric}| \leq K$ must satisfy $|Rm| \leq Ct^{-1}$ for all $t \in (0, T]$, where $C$ depends only on the dimension $n$, and $T$ depends on $K$ and the injectivity radius $\operatorname{inj}_{g(t)}$.

In the second part of this paper, we discuss the behavior of Ricci curvature and its derivative when the injectivity radius is thoroughly unknown. In particular, another Shi-type estimate for Ricci curvature is derived when the derivative of Ricci curvature is controlled by the derivative of scalar curvature.

**Mathematics Subject Classification (2010):** 53C44 (primary); 58J05 (secondary).