

Synchronization is full measure for all α -deformations of an infinite class of continued fractions

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Abstract. We study an infinite family of one-parameter deformations, called α -continued fractions, of interval maps associated to distinct triangle Fuchsian groups. In general for such one-parameter deformations, the function giving the entropy of the map indexed by α varies in a way directly related to whether or not the orbits of the endpoints of the map synchronize. For Nakada's original α -continued fractions and for certain continued fractions introduced by Katok-Ugarcovici, both of which are associated to the classical case of the modular group $\mathrm{PSL}_2(\mathbb{Z})$, the full parameter set for which synchronization occurs has been determined.

Here, we explicitly determine the synchronization sets for each α -deformation in our infinite family. (In general, our Fuchsian groups are not subgroups of the modular group, and hence the tool of relating α -expansions back to regular continued fraction expansions is not available to us.) A curiosity here is that all of our non-synchronization sets can be described in terms of a single tree of words. In a paper in preparation, we apply the results of this present work so as to find planar extensions of each of the maps, and thereby study the entropy functions associated to each deformation. We give an indication of this in the final section here.

Mathematics Subject Classification (2010): 11K50 (primary); 37A10, 37E05 (secondary).