

Integrated semigroups and parabolic equations Part II: semilinear problems

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Abstract. In this note we study a class of non-autonomous semilinear abstract Cauchy problems involving non-densely defined almost sectorial operator. The nonlinearity may contain unbounded terms and acts on suitable fractional power spaces associated with the almost sectorial operator. We use the framework of the so-called integrated semigroups to investigate the well-posedness of the problems. This note is a continuation of a previous work [9] dealing with linear equations. Here, using a suitable notion of mild solutions, we first study the existence of a maximal and strongly continuous evolution semiflow for semilinear equations under rather mild assumptions. Under additional conditions we prove that the semiflow is Fréchet differentiable and state some consequences about the linear stability of equilibria. In addition we prove that the solutions become immediately smooth so that the mild solutions turn out to be classical. We complete this work with an application of the results presented in this note to a reaction-diffusion equation with nonlinear and nonlocal boundary conditions arising, in particular, in mathematical biology.

Mathematics Subject Classification (2010): 47D06 (primary); 47D62, 35K90 (secondary).