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Free boundary minimal surfaces: a nonlocal approach

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Abstract. Given a C^k-smooth closed embedded manifold $\mathcal{N} \subset \mathbb{R}^m$, with $k \geq 2$, and a compact connected C^∞-smooth Riemannian surface (S, g) with $\partial S \neq \emptyset$, we consider $\frac{1}{2}$-*harmonic maps* $u \in H^{1/2}(\partial S, \mathcal{N})$. These maps are critical points of the nonlocal energy

$$E(f; g) := \int_S |\nabla \widetilde{u}|^2 \, d\mathrm{vol}_g, \tag{0.1}$$

where $\widetilde{u}$ is the harmonic extension of u in S. We express the energy (0.1) as a sum of the $\frac{1}{2}$-energies at each boundary component of ∂S (suitably identified with the circle $\mathcal{S}^1$), plus a quadratic term which is continuous in the $H^s(\mathcal{S}^1)$ topology, for any $s \in \mathbb{R}$. We show the $C^{k-1,\delta}$ regularity of $\frac{1}{2}$-harmonic maps. We also establish a connection between free boundary minimal surfaces and critical points of E with respect to variations of the pair (f, g), in terms of the Teichmüller space of S.

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