Isogenies of Abelian Anderson A-modules and A-motives

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Abstract. As a generalization of Drinfeld modules, Greg Anderson introduced Abelian t-modules and t-motives over a perfect base field. In this article we study relative versions of these defined over base rings. We investigate isogenies among them. Our main results state that every isogeny possesses a dual isogeny in the opposite direction, and that a morphism between Abelian t-modules is an isogeny if and only if the corresponding morphism between their associated t-motives is an isogeny. We also study torsion submodules of Abelian t-modules which in general are non-reduced group schemes. They can be obtained from the associated t-motive via the finite shtuka correspondence of Drinfeld and Abrashkin. The inductive limits of torsion submodules are the function field analogs of p-divisible groups. These limits correspond to the local shtukas attached to the t-motives associated with the Abelian t-modules. In this sense the theory of Abelian t-modules is captured by the theory of t-motives.