A spectral characterization of geodesic balls in non-compact rank one symmetric spaces

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Abstract. In constant curvature spaces, there are many characterizations of geodesic balls as optimal domain for shape optimization problems. Although it is natural to expect similar characterizations in rank one symmetric spaces, very little is known in this setting.

In this paper we prove that, in a non-compact rank one symmetric space, the geodesic balls uniquely maximize the first nonzero Steklov eigenvalue among the domains of fixed volume, extending to this context a result of Brock in the Euclidean space. Then we show that a stability version of the ensuing Brock-Weinstock inequality holds. The idea behind the proof is to exploit a suitable weighted isoperimetric inequality which we prove to hold true, as well as in a stability form, on harmonic manifolds.

Eventually we show that, in general, the geodesic balls are not global maximizers on the standard sphere.

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