

## A second order minimality condition for a free-boundary problem

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**Abstract.** The goal of this paper is to derive in the two-dimensional case necessary and sufficient minimality conditions in terms of the second variation for the functional

$$v \mapsto \int_{\Omega} \left( |\nabla v|^2 + \chi_{\{v>0\}} Q^2 \right) dx,$$

introduced in a classical paper of Alt and Caffarelli. For a special choice of  $Q$  this includes water waves. The second variation is obtained by computing the second derivative of the functional along suitable variations of the free boundary. It is proved that the strict positivity of the second variation gives a sufficient condition for local minimality. Also, it is shown that smooth critical points are local minimizers in a small tubular neighborhood of the free-boundary.

**Mathematics Subject Classification (2010):** 35R35 (primary); 49J40 (secondary).