

On the Picard numbers of Abelian varieties

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Abstract. In this paper we study the possible Picard numbers ρ of an Abelian variety A of dimension g . It is well known that this satisfies the inequality $1 \leq \rho \leq g^2$. We prove that the set R_g of realizable Picard numbers of Abelian varieties of dimension g is not complete for every $g \geq 3$, namely that $R_g \subsetneq [1, g^2] \cap \mathbb{N}$. Moreover, we study the structure of R_g as $g \rightarrow +\infty$, and from that we deduce a structure theorem for Abelian varieties of large Picard number. In contrast to the non-completeness of any of the sets R_g for $g \geq 3$, we also show that the Picard numbers of Abelian varieties are asymptotically complete, *i.e.*, $\lim_{g \rightarrow +\infty} \#R_g/g^2 = 1$. As a byproduct, we deduce a structure theorem for Abelian varieties of large Picard number. Finally we show that all realizable Picard numbers in R_g can be obtained by an Abelian variety defined over a number field.

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