

Del Pezzo elliptic varieties of degree $d \leq 4$

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Abstract. Let Y be a smooth del Pezzo variety of dimension $n \geq 3$, *i.e.*, a smooth complex projective variety endowed with an ample divisor H such that $-K_Y = (n-1)H$. Let d be the degree H^n of Y and assume that $d \leq 4$. Consider a linear subsystem of $|H|$ whose base locus is zero-dimensional of length d . The subsystem defines a rational map onto \mathbb{P}^{n-1} and, under some mild extra hypothesis, the general pseudofibers are elliptic curves. We study the elliptic fibration $X \rightarrow \mathbb{P}^{n-1}$ obtained by resolving the indeterminacy and call the variety X a del Pezzo elliptic variety. Extending the results of [7] we mainly prove that the Mordell-Weil group of the fibration is finite if and only if the Cox ring of X is finitely generated.

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