

A Poincaré map for the horocycle flow on $\mathrm{PSL}(2, \mathbb{Z}) \backslash \mathbb{H}$ and the Stern-Brocot tree

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Abstract. We construct a Poincaré map $\mathcal{P}_h$ for the positive horocycle flow on the modular surface $\mathrm{PSL}(2, \mathbb{Z}) \backslash \mathbb{H}$, and begin a systematic study of its dynamical properties. In particular, we give a complete characterisation of the periodic orbits of $\mathcal{P}_h$, we show that they are equidistributed with respect to the invariant measure of $\mathcal{P}_h$, and we organise them using the Stern-Brocot tree of rational numbers. In addition, we introduce a time-reparameterisation of $\mathcal{P}_h$ which gives an insight into the dynamics of the non-periodic orbits.

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