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Ordinary isogeny graphs over $\mathbb{F}_p$: the inverse volcano problem

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Abstract. We give a detailed presentation of ℓ-isogeny graphs associated with ordinary elliptic curves defined over $\mathbb{F}_p$. We then focus on the following inverse problem: given an abstract volcano V, can one always find primes ℓ, $p \in \mathbb{N}$ such that the ordinary ℓ-isogeny graph over $\mathbb{F}_p$ contains V as a connected component? We provide an answer to both this and a stronger version of the question, where all the vertices of the crater of the abstract volcano are required to correspond to elliptic curves with complex multiplication by a maximal order. To this aim, we generalise a classical theorem by Yamamoto on orders of prime ideals in the class group of imaginary quadratic fields.

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