

Regularity of almost-surely injective projections in Euclidean spaces

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Abstract. In a previous work we proved that, if a finite Borel measure μ in a Euclidean space has Hausdorff dimension smaller than a positive integer k , then the orthogonal projection onto almost every k -dimensional linear subspace is injective on a set of full μ -measure. In this paper we study the regularity of the inverses of these projections and prove that, if μ has a compact support X such that (respectively) the Hausdorff, the upper box-counting, or the Assouad dimension of X is smaller than k , then the inverse is (respectively) continuous, pointwise α -Hölder for some $\alpha \in (0, 1)$, or pointwise α -Hölder for every $\alpha \in (0, 1)$. These results generalize to the case of typical linear perturbations of Lipschitz maps and strengthen previously known ones in the lossless analog compression literature. We provide examples showing sharpness of the statements. Additionally, we construct a non-trivial measure on the plane which admits almost-surely injective projections in every direction, and show that no homogeneous self-similar measure has this property.

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