

## Quantitative recurrence properties for piecewise expanding maps on $[0, 1]^d$

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**Abstract.** Let  $T: [0, 1]^d \rightarrow [0, 1]^d$  be a piecewise expanding map with an absolutely continuous invariant measure  $\mu$ . Let  $\{H_n\}$  be a sequence of hyperrectangles or hyperboloids centered at the origin. Denote by  $\mathcal{R}(\{H_n\})$  the set of points  $\mathbf{x} \in [0, 1]^d$  such that  $T^n \mathbf{x} \in \mathbf{x} + H_n$  for infinitely many  $n \in \mathbb{N}$ , where  $\mathbf{x} + H_n$  is the translation of  $H_n$ . We prove that, if  $\mu$  is exponential mixing and the density of  $\mu$  is sufficiently regular, then the  $\mu$ -measure of  $\mathcal{R}(\{H_n\})$  is zero or full depending on whether the sum of the volumes of  $H_n$  converges or not. In the case where  $T$  is a matrix transformation, our results extend a previous work of Kirsebom, Kunde, and Persson in two aspects: by allowing the matrix to be non-integer and by allowing the ‘targets’  $H_n$  to be hyperrectangles or hyperboloids. We also obtain a dimension result when  $T$  is a diagonal matrix transformation.

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