

Symmetry for a fully nonlinear free boundary problem with highly a singular term

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Abstract. In this paper, we establish the radial symmetry of solutions to a free-boundary problem, characterized by a singular right-hand side, both in the elliptic and parabolic regimes. Specifically, we focus on the unit ball B_1 of $\mathbb{R}^n$, where we examine a solution to the fully nonlinear elliptic problem given by:

$$\begin{cases} F(D^2u) = f(u) & \text{in } B_1 \cap \{u > 0\} \\ u = M & \text{on } \partial B_1 \\ 0 \leq u < M & \text{in } B_1. \end{cases}$$

The right-hand side $f(u)$ behaves near the free boundary $\partial\{u > 0\}$ like u^a , with negative values for $a \in (-1, 0)$. It is important to note that due to the lack of C^2 -smoothness of both u and the free boundary $\partial\{u > 0\}$, the well-known Serrin-type boundary point lemma cannot be directly applied. To overcome this challenge, we devise an approach based on an exact assumption concerning the first-order expansion and the decay on the second order, complemented by an ad-hoc comparison principle. Furthermore, we extend our analysis to encompass the parabolic case of the problem, and present a corresponding result.

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