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Varieties of groups and the problem on conciseness of words

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Abstract. A group-word w is concise in a class of groups $\mathcal{X}$ if and only if the verbal subgroup $w(G)$ is finite whenever w takes only finitely many values in a group $G \in \mathcal{X}$. It is a long-standing open problem whether every word is concise in residually finite groups. In this paper we observe that the conciseness of a word w in residually finite groups is equivalent to that in the class of virtually pro-p groups. This is used to show that if q, n are positive integers and w is a multilinear commutator word, then the words w^q and $[w^q,_n y]$ are concise in residually finite groups. Earlier this was known only in the case where q is a prime power. In the course of the proof we establish that certain classes of groups satisfying the law $w^q \equiv 1$, or $[\delta_k^q,_n y] \equiv 1$, are varieties.

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