Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5)
Vol. IV (2005), 129-177

Minimal surfaces in pseudohermitian geometry

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Abstract. We consider surfaces immersed in three-dimensional pseudohermitian manifolds. We define the notion of (p-)mean curvature and of the associated (p-)minimal surfaces, extending some concepts previously given for the (flat) Heisenberg group. We interpret the p-mean curvature not only as the tangential sublaplacian of a defining function, but also as the curvature of a characteristic curve, and as a quantity in terms of calibration geometry.
As a differential equation, the p-minimal surface equation is degenerate (hyperbolic and elliptic). To analyze the singular set (*i.e.*, the set where the (p-)area integrand vanishes), we formulate some *extension* theorems, which describe how the characteristic curves meet the singular set. This allows us to classify the entire solutions to this equation and to solve a Bernstein-type problem (for graphs over the xy-plane) in the Heisenberg group H_1. In H_1, identified with the Euclidean space $\mathbb{R}^3$, the p-minimal surfaces are classical ruled surfaces with the rulings generated by Legendrian lines. We also prove a uniqueness theorem for the Dirichlet problem under a condition on the size of the singular set in two dimensions, and generalize to higher dimensions without any size control condition.
We also show that there are no closed, connected, C^2 smoothly immersed constant p-mean curvature or p-minimal surfaces of genus greater than one in the standard S^3. This fact continues to hold when S^3 is replaced by a general pseudohermitian 3-manifold.

Mathematics Subject Classification (2000): 35L80 (primary); 35J70, 32V20, 53A10, 49Q10 (secondary).