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On the concave one-dimensional random assignment problem and Young's integration theory

MICHAEL GOLDMAN AND DARIO TREVISAN

Dedicated to L. Ambrosio on his 60th birthday

Abstract. We investigate the one-dimensional random assignment problem in the concave case, *i.e.*, with the assignment cost being a concave power function, with exponent $0 < p < 1$, of the distance between n source and n target points, that are i.i.d. random variables with a common law on an interval. We prove that the limit of a suitable renormalization of the cost exists if the exponent p is different from $1/2$. Our proof in the case $1/2 < p < 1$ makes use of a novel version of the Kantorovich optimal transport theory, based on Young's integration theory, where the difference between two measures is replaced by the weak derivative of a function with finite q-variation, which may be of independent interest. We also prove a similar result for the random bipartite traveling salesperson problem.

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