

Some relations among volume, intrinsic perimeter and one-dimensional restrictions of BV functions in Carnot groups

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Abstract. Let \mathbb{G} be a k -step Carnot group. The first aim of this paper is to show an interplay between volume and \mathbb{G} -perimeter, using one-dimensional horizontal slicing. What we prove is a kind of Fubini theorem for \mathbb{G} -regular submanifolds of codimension one. We then give some applications of this result: slicing of $BV_{\mathbb{G}}$ functions, integral geometric formulae for volume and \mathbb{G} -perimeter and, making use of a suitable notion of convexity, called \mathbb{G} -convexity, we state a Cauchy type formula for \mathbb{G} -convex sets. Finally, in the last section we prove a sub-Riemannian Santaló formula showing some related applications. In particular we find two lower bounds for the first eigenvalue of the Dirichlet problem for the Carnot sub-Laplacian $\Delta_{\mathbb{G}}$ on smooth domains.

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