Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXVII (2026), 85-107

On unexpected curves of type $(d+k, d)$

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Abstract. We present a construction explaining the existence of (unexpected) curves of degree $d+k$, passing through a finite set Z of points on $\mathbb{P}^2$, and having a generic point P of multiplicity d. The construction is based on the syzygies of the k-th powers of the Jacobian of the product of lines dual to the points of Z. These syzygies give rise to a vector bundle of rank $k+1$. We prove a result giving a sufficient condition for the unexpectedness of curves via the splitting type of the bundle (restricted to the line dual to P) providing a generalization of the theory initiated by Faenzi and Valles and by Cook II, Harbourne, Migliore and Nagel.

Mathematics Subject Classification (2020): 14N20 (primary); 14C20 (secondary).