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Weak and strong L^p-limits of vector fields with finitely many integer singularities in dimension n

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Abstract. For every given $p \in [1, +\infty)$ and $n \in \mathbb{N}$ with $n \geqslant 1$, the authors identify the strong L^p-closure $L^p_{\mathbb{Z}}(D)$ of the class of vector fields having finitely many integer topological singularities on a domain D which is either bi-Lipschitz equivalent to the open unit n-dimensional cube or to the boundary of the unit $(n+1)$-dimensional cube. Moreover, for any $n \in \mathbb{N}$ with $n \geqslant 2$ the authors prove that $L^p_{\mathbb{Z}}(D)$ is weakly sequentially closed for every $p \in (1, +\infty)$ whenever D is an open domain in $\mathbb{R}^n$ which is bi-Lipschitz equivalent to the open unit cube. As a byproduct of our analysis, a useful characterisation of such vector fields is obtained in terms of existence of a (minimal) connection for their singular set.

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