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# Divides and hyperbolic volumes

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**Abstract.** A divide is the image of a proper and generic immersion of a compact 1-manifold into the 2-disk. Due to A'Campo's theory, to each divide one can associate a link in the 3-sphere. In this paper, we reveal a hidden hyperbolic structure in the theory of links of divides. More precisely, we show that the complement of the link of a divide can be obtained by Dehn-filling a hyperbolic 3-manifold that admits a decomposition into several ideal regular tetrahedra, octahedra and cuboctahedra, where the number of each of these polyhedra is determined by the types of the double points of the divide. This immediately gives an upper bound for the hyperbolic volume of the links of divides, which is shown to be asymptotically sharp. An idea from Turaev's theory of shadows plays an important role here.

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