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On the transcendence of a series related to Sturmian words

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Abstract. Let b be an algebraic number with $|b| > 1$ and $\mathcal{H}$ be a finite set of algebraic numbers. We study the transcendence of numbers of the form $\sum_{n=0}^{\infty} \frac{a_n}{b^n}$, where $a_n \in \mathcal{H}$ for all $n \in \mathbb{N}$. We assume that the sequence $(a_n)_{n=0}^{\infty}$ is generated by coding the orbit of a point under an irrational rotation of the unit circle. In particular, this assumption holds whenever the sequence is Sturmian. Our main result shows that, apart from some trivial exceptions, all numbers of the above form are transcendental. We moreover give sufficient conditions for a finite set of such numbers to be linearly independent over $\overline{\mathbb{Q}}$.

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