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Smooth projective surfaces with infinitely many real forms

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Abstract. The aim of this paper is twofold. First of all, we confirm a few basic finiteness for the criteria real forms of a given smooth complex projective variety, in terms of the Galois cohomology set of the discrete part of the automorphism group, the cone conjecture and the topological entropy. We then apply these criteria to show that a smooth complex projective surface has at most finitely many non-isomorphic real forms unless it is either rational or a non-minimal surface birational to either a K3 surface or an Enriques surface. In the second part of the paper, we construct an Enriques surface whose blow-up at one point admits infinitely many non-isomorphic real forms. This answers a question of Kondo, and shows that the three exceptional cases of the classification all occur.

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