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Well-posedness results for hyperbolic operators with coefficients oscillating in time

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Abstract. In the present paper, we consider second-order strictly hyperbolic linear operators of the form $Lu = \partial_t^2 u - \operatorname{div}\big(A(t,x)\nabla u\big)$, for $(t,x) \in [0,T] \times \mathbb{R}^n$. We assume the coefficients of the matrix $A = A(t,x)$ to be smooth in time on $]0,T] \times \mathbb{R}^n$, but oscillating when $t \to 0^+$; they match instead minimal regularity assumptions (either Lipschitz or log-Lipschitz regularity conditions) with respect to the space variable.

Correspondingly, we prove well-posedness results for the Cauchy problem related to L, either with no loss of derivatives (in the Lipschitz case) or with a finite loss of derivatives, which is linearly increasing in time (in the log-Lipschitz case).

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