

Higgs bundles and flat connections over compact Sasakian manifolds, II: quasi-regular bundles

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Abstract. In this continuation of [7] and [8] we investigate the non-Abelian Hodge correspondence on compact Sasakian manifolds with emphasis on the quasi-regular case. We introduce on quasi-regular Sasakian manifolds the notions of quasi-regularity and regularity of basic vector bundles. These notions are useful in relating the vector bundles over a quasi-regular Sasakian manifold with the orbibundles over the orbifold defined by the orbits of the Reeb foliation of the quasi-regular Sasakian manifold. We note that the non-Abelian Hodge correspondence on any quasi-regular Sasakian manifold gives a canonical correspondence between the semisimple representations of the orbifold fundamental group and the Higgs orbibundles on a locally cyclic complex orbifold admitting a Hodge metrics. Under the assumption of quasi-regularity of Sasakian manifolds and vector bundles, we extend this correspondence to one between the flat bundles and the basic Higgs bundles. We also prove a Sasakian analogue of the characterization of numerically flat bundles given by Demailly, Peternell and Schneider.

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