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Ergodic properties of a parameterised family of symmetric golden maps: the matching phenomenon revisited

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Abstract. We study a one-parameter family of interval maps $\{T_\alpha\}_{\alpha\in[1,\beta]}$, with golden mean β the defined on $[-1,1]$ by $T_\alpha(x) = \beta^{1+|t|}x - t\beta\alpha$, where $t \in \{-1,0,1\}$ is determined piecewise. For each $T_\alpha,\ \alpha > 1$, we construct its unique, absolutely continuous invariant measure and show that on an open, dense subset of parameters α, the corresponding density is a step function with finitely many jumps. We give an explicit description of the maximal intervals of parameters on which the density has at most the same number of jumps. A main tool in our analysis is the phenomenon of matching, where the orbits of the left and right limits of discontinuity points meet after a finite number of steps. Each T_α generates signed expansions of numbers in base $1/\beta$; via Birkhoff's ergodic theorem, the invariant measures are used to determine the asymptotic relative frequencies of digits in generic T_α-expansions. In particular, the frequency of 0 is shown to vary continuously as a function of α and to attain its maximum $3/4$ on the maximal interval $[1/2 + 1/\beta, 1 + 1/\beta^2]$.

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