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Spectral analysis for some third-order differential equations: a semigroup approach

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Abstract. In this paper we consider the ordinary differential equation of third order

$$\frac{d^3u}{dt^3} + A^{\theta}\frac{d^2u}{dt^2} + A^{\varrho}\frac{du}{dt} + Au = 0,$$

where $0 \leqslant \theta < \varrho \leqslant 1$, subject to initial conditions

$$u(0) = u_0, \quad \frac{du}{dt}\Big|_{t=0}(0) = u_1, \quad \frac{d^2u}{dt^2}\Big|_{t=0}(0) = u_2,$$

where X is a separable Hilbert space, $A : D(A) \subset X \to X$ is an unbounded, linear, closed, densely-defined, self-adjoint and positive definite operator, and $A^{\alpha} : D(A^{\alpha}) \subset X \to X$ denotes the fractional power of A for $\alpha \in (0, 1)$. We discuss the possibility of these problems being well posed (here well-posedness means that there is a strongly continuous semigroup associated to the equation) in a suitable phase space and for different choices of $0 \leqslant \theta < \varrho \leqslant 1$; namely, $\theta + \varrho < 1$ (range for which the problem is not well posed), $\theta + \varrho = 1$ (range for which the problem may be well posed) and $\theta + \varrho > 1$ (range where the problem may be well posed and the associated semigroup may be analytic). Moreover, we present an application of our results for evolutionary equations.

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