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Gradient flows in asymmetric metric spaces and applications

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Abstract. This paper is devoted to the investigation of gradient flows in asymmetric metric spaces (for example, irreversible Finsler manifolds and Minkowski normed spaces) by means of discrete approximation. We study basic properties of curves and upper gradients in asymmetric metric spaces, and establish the existence of a curve of maximal slope, which is regarded as a gradient curve in the non-smooth setting. Introducing a natural convexity assumption on the potential function, which is called the (p,λ)-convexity, we also obtain some regularizing effects on the asymptotic behavior of curves of maximal slope. Applications include several existence results for gradient flows in Finsler manifolds, doubly nonlinear differential evolution equations on infinite-dimensional Funk spaces, and heat flows on compact Finsler manifolds.

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