Hardy spaces, Besov spaces and Triebel–Lizorkin spaces associated with a discrete Laplacian and applications

THE ANH BUI AND XUAN THINH DUONG

Abstract. Consider the discrete Laplacian Δ_d defined on the set of integers $\mathbb{Z}$ by

$$\Delta_d f(n) = -f(n+1) + 2f(n) - f(n-1), \quad n \in \mathbb{Z},$$

where f is a function defined on $\mathbb{Z}$. In this paper, we define Hardy spaces, Besov spaces and Triebel–Lizorkin spaces associated with Δ_d and then show that these function spaces coincide with the classical function spaces defined on $\mathbb{Z}$. As applications, we prove the boundedness of the spectral multipliers and the Riesz transforms associated with Δ_d on these function spaces.

Mathematics Subject Classification (2020): 39A12 (primary); 35K08, 42A38, 42B35, 42B25, 42B15 (secondary).