

Uniqueness and weak stability for multi-dimensional transport equations with one-sided Lipschitz coefficient

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Abstract. The Cauchy problem for a multidimensional linear transport equation with discontinuous coefficient is investigated. Provided the coefficient satisfies a one-sided Lipschitz condition, existence, uniqueness and weak stability of solutions are obtained for either the conservative backward problem or the advective forward problem by duality. Specific uniqueness criteria are introduced for the backward conservation equation since weak solutions are not unique. A main point is the introduction of a generalized flow in the sense of partial differential equations, which is proved to have unique Jacobian determinant, even though it is itself nonunique.

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