Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XVIII (2018), 781-829

Symplectic Wick rotations between moduli spaces of 3-manifolds

CARLOS SCARINCI AND JEAN-MARC SCHLENKER

Abstract. We describe natural maps between (parts of) $\mathcal{QF}$, the space of quasi-fuchsian hyperbolic metrics on a product 3-manifold $S \times \mathbb{R}$, and $\mathcal{GH}_{-1}$, the space of maximal globally hyperbolic anti-de Sitter metrics on the same manifold, defined in terms of special surfaces (*e.g.*, minimal/maximal surfaces, CMC surfaces, pleated surfaces) and prove that these "Wick rotations" are at least C^1 smooth and symplectic with respect to the canonical symplectic structures on both $\mathcal{QF}$ and $\mathcal{GH}_{-1}$. Similar results involving the spaces of globally hyperbolic de Sitter and Minkowski metrics are also described.

These 3-dimensional results are shown to be equivalent to purely 2-dimensional ones. Namely, consider the double harmonic map $\mathcal{H} : T^*\mathcal{T} \to \mathcal{T} \times \overline{\mathcal{T}}$, sending a conformal structure c and a holomorphic quadratic differential q on S to the pair of hyperbolic metrics (m_L, m_R) such that the harmonic maps isotopic to the identity from (S, c) to (S, m_L) and to (S, m_R) have, respectively, Hopf differentials equal to iq and $-iq$, and the double earthquake map $\mathcal{E} : \mathcal{T} \times \mathcal{ML} \to \mathcal{T} \times \overline{\mathcal{T}}$, sending a hyperbolic metric m and a measured lamination l on S to the pair $(E_L(m, l), E_R(m, l))$, where E_L and E_R denote the left and right earthquakes. We describe how such 2-dimensional double maps are related to 3-dimensional Wick rotations and prove that they are also C^1 smooth and symplectic.

Mathematics Subject Classification (2010): 53C35 (primary); 53C50, 53C40, 83C80 (secondary).