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Isometries of the space of Sasaki potentials

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Abstract. Given any two Kähler manifolds X_1 and X_2, L. Lempert recently proved that if their spaces of Kähler potentials are isometric with respect to the Mabuchi metric, then X_1 and X_2 must be diffeomorphic. We prove that this is no longer the case for Sasaki manifolds. Then, considering *regular* Sasaki manifolds M_1 and M_2, we prove that if the spaces of potentials are isometric, then M_1 and M_2 must have, among other things, the same universal covering space. Finally, getting rid of the regularity assumption on M_1 and M_2, we investigate the consequences of the existence of *affine* Mabuchi isometries: this leads to a family of Sasaki isospectral structures.

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