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Rational blowdown graphs for symplectic fillings of lens spaces

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Abstract. In a previous work, we proved that each minimal symplectic filling of any oriented lens space, viewed as the singularity link of some cyclic quotient singularity and equipped with its canonical contact structure, can be obtained from the minimal resolution of the singularity by a sequence of symplectic rational blowdowns along linear plumbing graphs. Here we give a dramatically simpler visual presentation of our rational blowdown algorithm in terms of the triangulations of a convex polygon. As a consequence, we are able to organize the symplectic deformation equivalence classes of all minimal symplectic fillings of any given lens space equipped with its canonical contact structure as a graded, directed, rooted, and connected graph, where the root is the minimal resolution of the corresponding cyclic quotient singularity and each directed edge is a symplectic rational blowdown along an explicit linear plumbing graph. Moreover, we provide an upper bound for the *rational blowdown depth* of each minimal symplectic filling.

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