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The nonlocal fractional mean curvature flow of periodic graphs

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Abstract. We establish the well-posedness of the nonlocal fractional mean curvature flow of order $\alpha \in (0,1)$ for periodic graphs on $\mathbb{R}^n$ in all subcritical little Hölder spaces $\mathrm{h}^{1+\beta}(\mathbb{T}^n)$ with $\beta \in (0,1)$. Furthermore, we prove that if the solution is initially sufficiently close to its integral mean in $\mathrm{h}^{1+\beta}(\mathbb{T}^n)$, then it exists globally in time and converges exponentially fast towards a constant. The proofs rely on the reformulation of the equation as a quasilinear evolution problem, which is shown to be of parabolic type by a direct localization approach, and on abstract parabolic theories for such problems.

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