# Occurrence of gap for one-dimensional scalar autonomous functionals with one end-point condition

RAPHAËL CERF AND CARLO MARICONDA

**Abstract.** Let $L : \mathbb{R} \times \mathbb{R} \to [0, +\infty[ \, \cup \{+\infty\}$ be a Borel function. We consider the problem

$$\min F(y) = \int_0^1 L(y(t), y'(t)) \, dt : y(0) = 0, \ y \in W^{1,1}([0, 1]). \quad \text{(P)}$$

We give an example of a real-valued Lagrangian $L$ for which the Lavrentiev phenomenon occurs. We state a condition, involving only the behavior of $L$ on the graph of two functions, that ensures the non-occurrence of the phenomenon. Our criterium substantially weakens the well-known condition that $L$ is bounded on bounded sets.

**Mathematics Subject Classification (2020):** 49N60 (primary); 39B15 (secondary).