

Blowing-up solutions for a nonlocal Liouville-type equation

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Abstract. We consider the nonlocal Liouville-type equation:

$$(-\Delta)^{\frac{1}{2}}u = \varepsilon\kappa(x)e^u, \quad u > 0 \quad \text{in } I, \quad u = 0 \quad \text{in } \mathbb{R} \setminus I,$$

where I is a union of $d \geq 2$ disjoint bounded intervals, κ is a smooth bounded function with positive infimum and $\varepsilon > 0$ is a small parameter. For any integer $1 \leq m \leq d$, we construct a family of solutions $(u_\varepsilon)_\varepsilon$ which blow up at m interior distinct points of I and for which $\varepsilon \int_I \kappa e^{u_\varepsilon} \rightarrow 2m\pi$, as $\varepsilon \rightarrow 0$. Moreover, we show that, when $d = 2$ and m is suitably large, no such construction is possible.

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