

## On the construction of $k$ -regular maps to Grassmannians via algebras of socle dimension two

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**Abstract.** A continuous map  $\mathbb{C}^n \rightarrow \text{Gr}(\tau, N)$  is  $k$ -regular if the  $\tau$ -dimensional subspaces corresponding to images of any  $k$  distinct points span a  $\tau k$ -dimensional space. For  $\tau = 1$  this essentially recovers the classical notion of a  $k$ -regular map  $\mathbb{C}^n \rightarrow \mathbb{C}^N$ . We provide new examples of  $k$ -regular maps, both in the classical setting  $\tau = 1$  and for  $\tau \geq 2$ , where these are the first examples known.

Our methods come from algebraic geometry, following and generalizing [11]. The key and highly nontrivial part of the argument is proving that certain loci of the Hilbert scheme of points have the expected dimension. As an important side result, we prove the irreducibility of the punctual Hilbert scheme of  $k$  points on a threefold, for  $k \leq 11$ .

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