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Fractional integration of summable functions: Maz'ya's Φ-inequalities

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Abstract. We study inequalities of the type $|\int_{\mathbb{R}^d} \Phi(K * f)| \lesssim \|f\|^p_{L_1(\mathbb{R}^d)}$, where the kernel K is homogeneous of order $\alpha - d$ and possibly vector-valued, the function Φ is positively p-homogeneous, and $p = d/(d-\alpha)$. Under mild regularity assumptions on K and Φ, we find necessary and sufficient conditions on these functions under which the inequality holds true with a uniform constant for all sufficiently regular functions f.

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